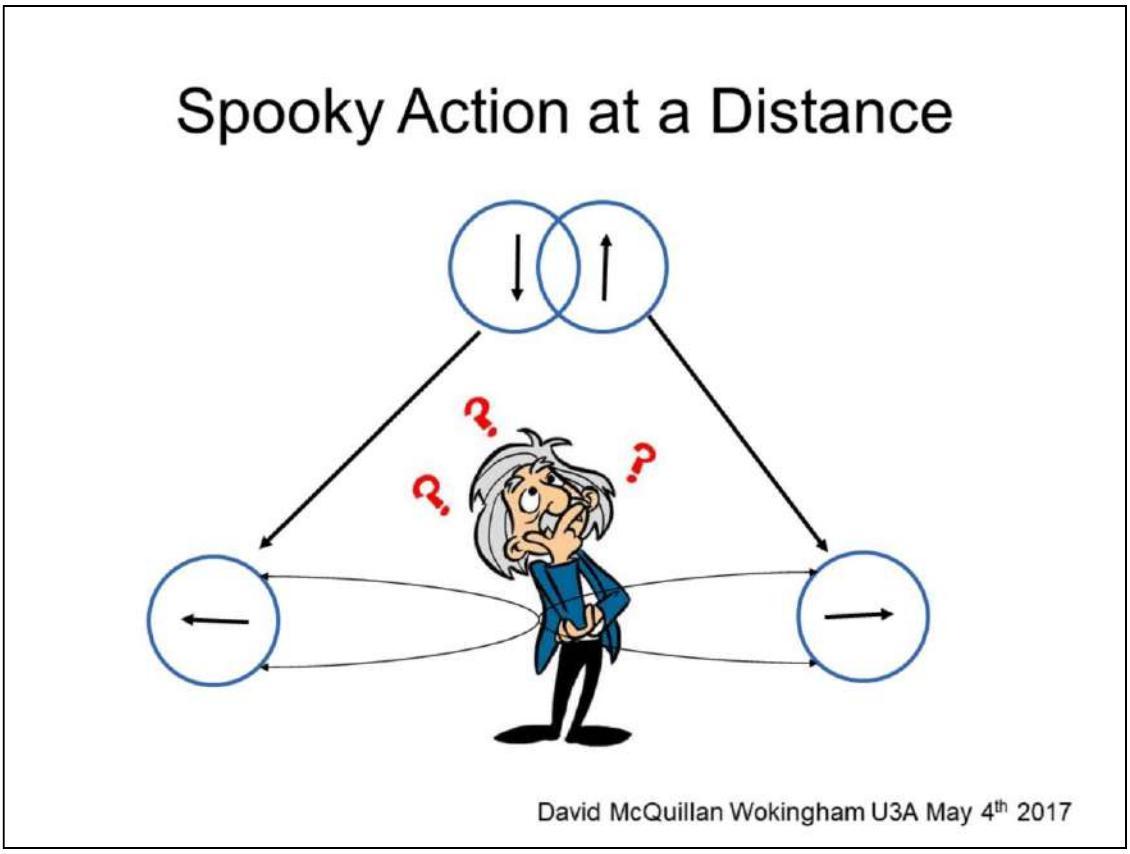
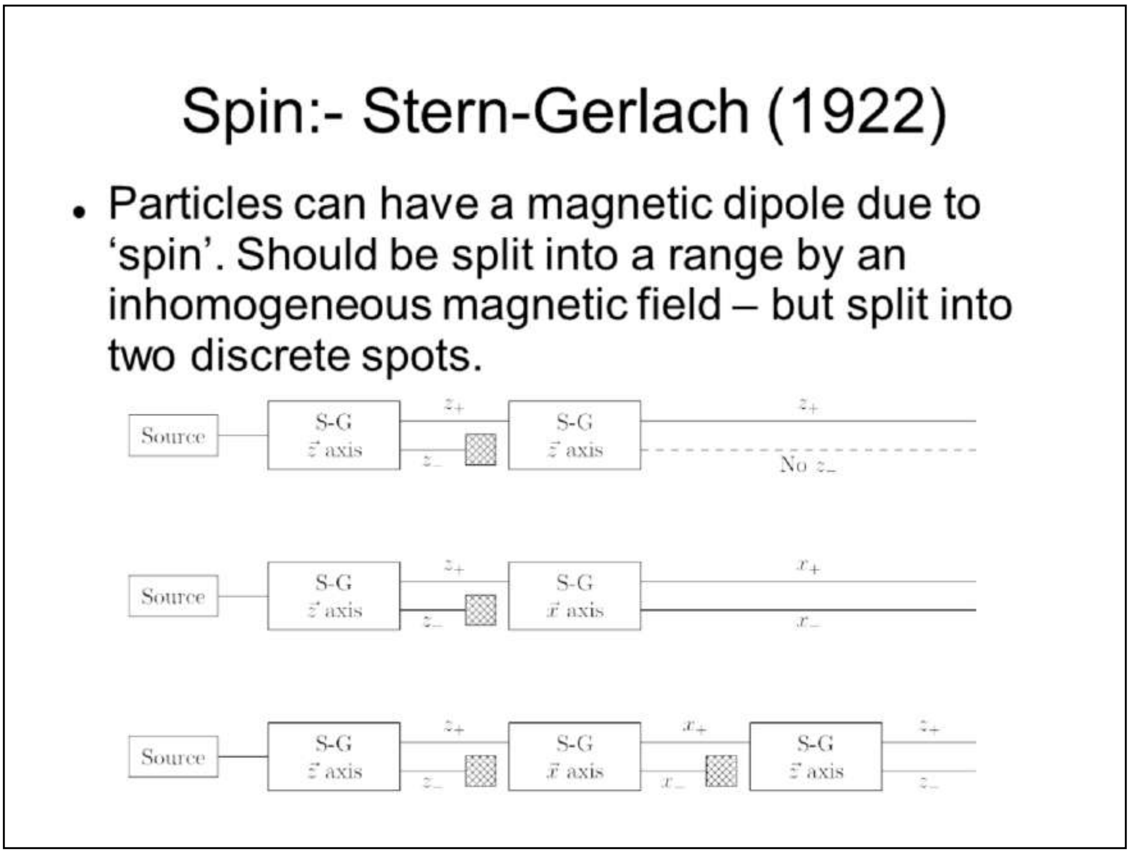


Newton was the first to wonder about how gravity could act over a distance without some intermediate effect but nobody came up with anything better till Einstein with General Relativity.

The term 'spooky action at a distance' famously used by Einstein to describe a simple thought experiment the Einstein–Podolsky–Rosen paradox devised in 1935 that seemed to require hidden variables in Quantum Mechanics.

The Plan

- Spin of a particle
- Entangled states
- Bell's Inequality
- Quantum states
- Measurement and reality
- Quantum pseudo-telepathy



Might think that the quantization was due to some interaction – but the rest of the talk will show this is not the case.

Neutral atoms of Silver used. Electrons would give a large deflection due to their charge. Want only an effect due to a gradient over a dipole.

Diagram from WIkipedia

What is Spin?

- Minimum spin quantum number s = 1/2
- Intrinsic angular momentum, same units as Planck constant

$$\hbar\sqrt{s(s+1)} = \frac{h}{2\pi}\sqrt{s(s+1)}$$

- Orbital angular momentum can only have values corrsponding to integer values of s
- Angular momentum is conserved

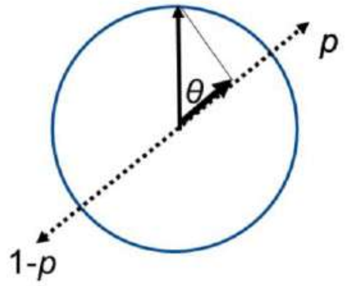
h is the Planck constant, the ‘quantum of action’. It is in units of angular momentum ML^2T^{-1}. It is about 6.62607×10−34 Joule-seconds

$\hbar$ is the reduced Planck constant or the Dirac constant $h/2\pi$.

Spin:- arbitrary angle

- Like the classical case on average.
- Angle θ overall spin $\cos\theta$
- Probability p in direction θ and 1-p in the opposite direction.

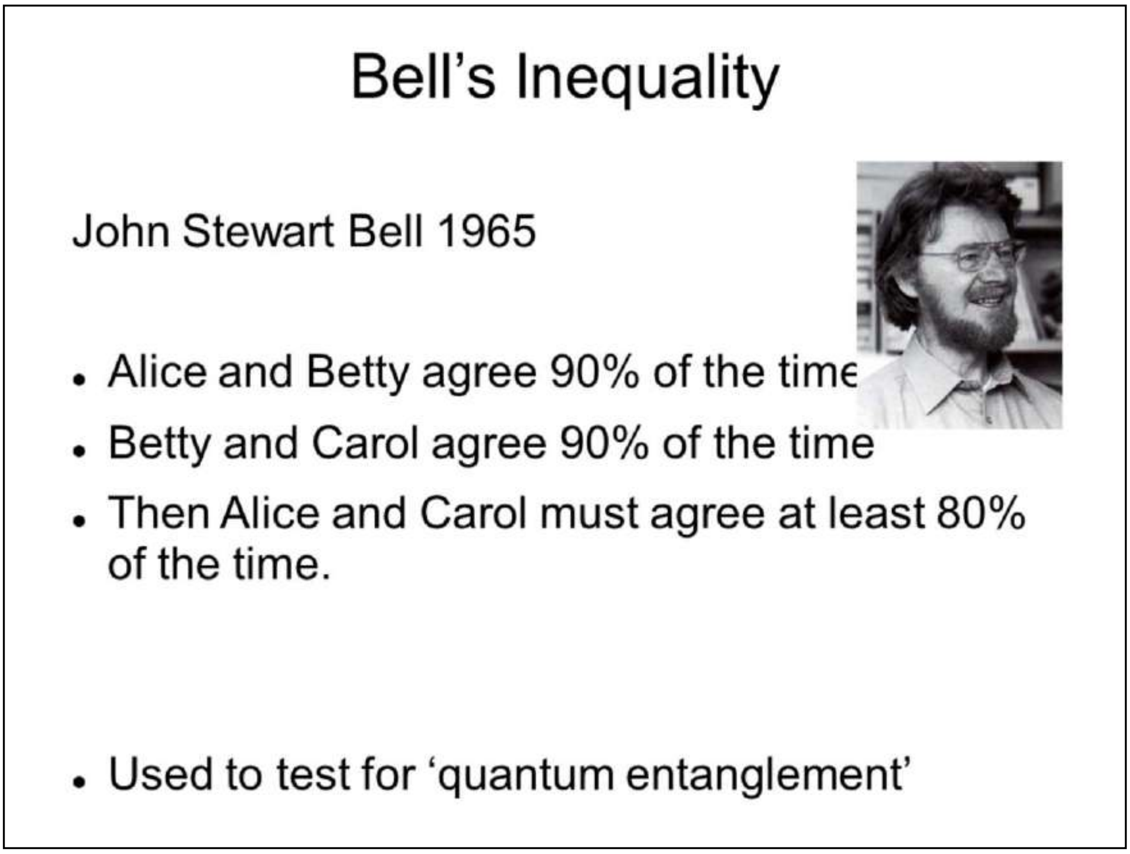
$$p + (-1) \times (1 - p) = \cos\theta$$



$$p = \frac{1 + \cos\theta}{2}$$

$$p = \cos^2\frac{\theta}{2}$$

And 1-p is sin squared theta over two which might strike a chord. We’ll come back to this interesting point later

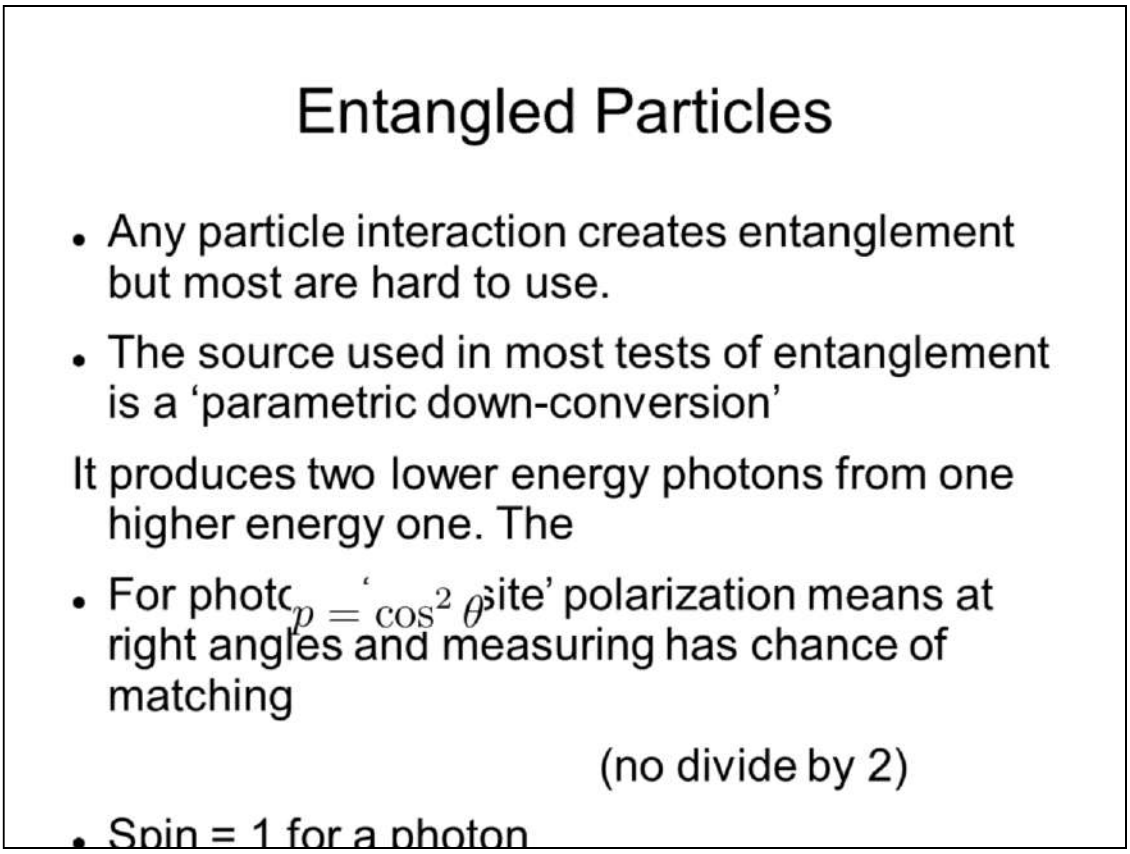


John Stewart Bell 1965

Well actually he said a lot more but this is good enough for here!

Bell's Theorem:

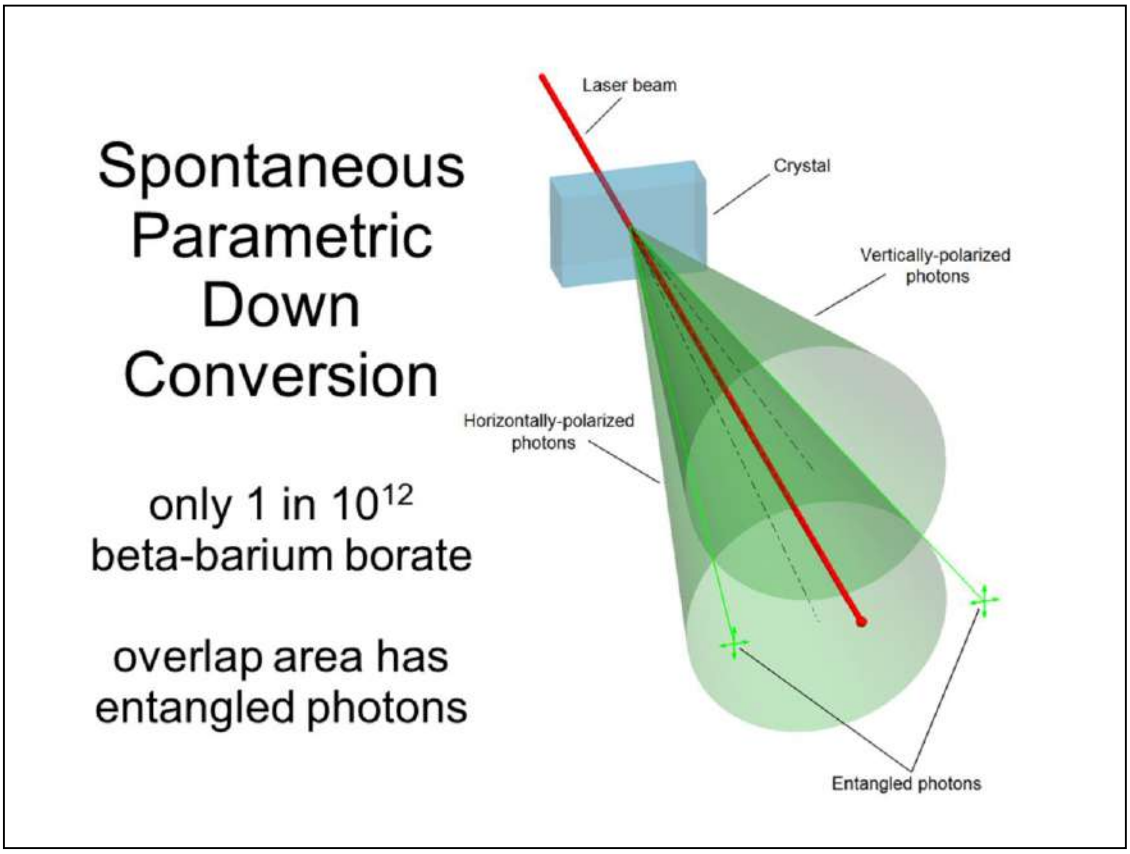
No physical theory of local hidden variables can ever reproduce all of the predictions of quantum mechanics.



It would be nice to use electrons but most of the experimental tests have used photons

Angular momentum and hence spin is conserved for entangled particles if nothing is done to them.

For spin 1 there are three possible values - -1, 0, +1 but the 0 case does not occur for photons which go at the speed of light

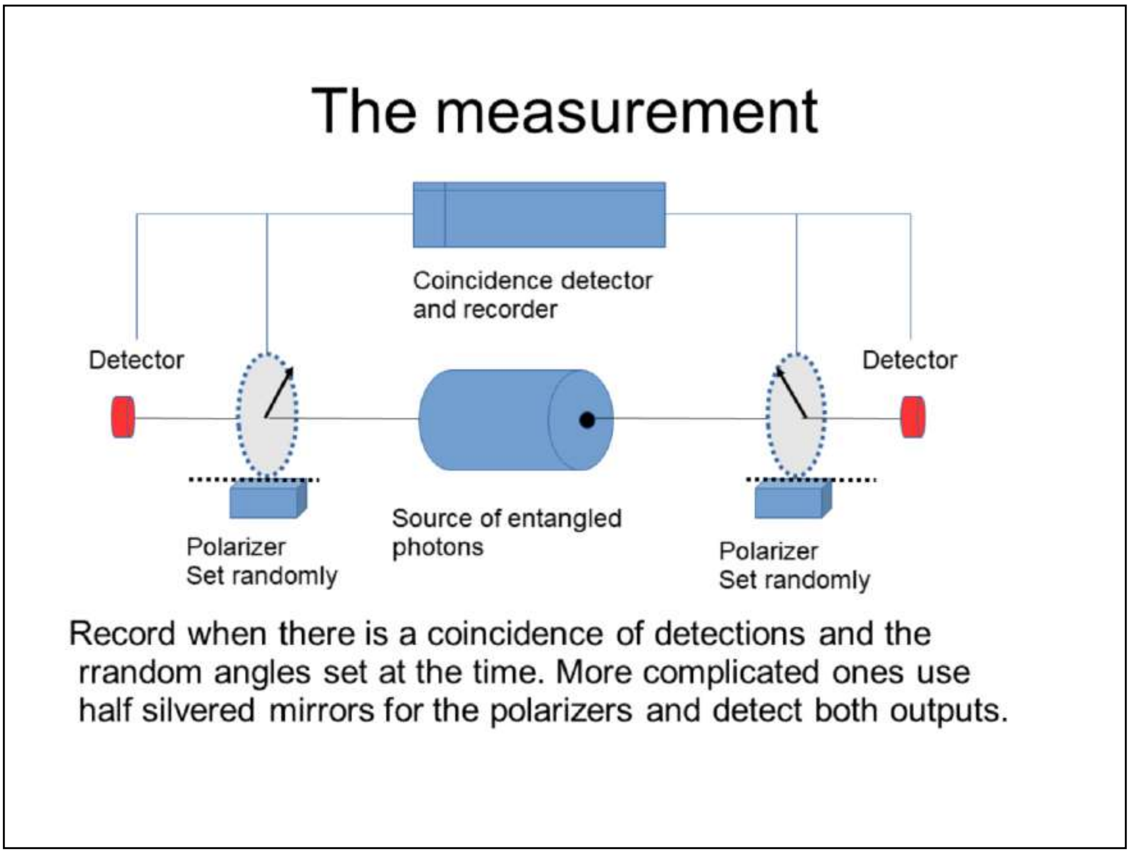


The photons come out in two rings

For entanglement we need output photons in the overlap region of the rings as there we don't know if they have one type of polarization or the other

Get one on either side due to conservation of momentum but selection may remove some of one or the other side.

They are a Bell pair, two particles with a fully entangled state. A measurement of one gives a completely random result, but it is completely correlated with the result from the same measurement for the other one.



Half silvered mirrors can do polarizaton and enable both options to be detected and counted.

Sent in opposite directions so the random change of the polarization can’t be sent to the other side before the random polarization and detection there.

Bell's Inequality for polarization

Angles here refer to the difference in polarization angles.

- A: 0° – probability 1
- B: 22.5° - $\cos^2 22.5°$ = 0.85355..
- C: 45° - probability 0.5

B to C should be the same as A to B

so p(A to C) should be >= 1-(1-p(A to B))+(1-p(B to C))

- However 0.5 is less than
  1-(1-0.85355)-(1-0.85355) = 0.70711

For light the verticl and horizontal polarizations are different spin states as opposed to up and down for an electron – but the marhs is just the same except for a factor of hald in te angle.

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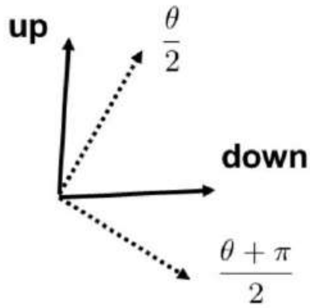
Quantum State

- State is a vector in a complex vector space of as many dimensions as there are independent possible measurements. For spin of an electron:-



Measurement Probabilities

$$\sin^2 \frac{\theta}{2} + \cos^2 \frac{\theta}{2} = 1$$

For a photon the diagram maps directly with theta instead of theta over two. And of course vertical and horizontal instead of up and down.

Electron spin state

- The state is described by a vector in a two dimensional complex space.
- The spin can be in any direction in 3D
- The state describes the two dimensional surface of a sphere as the norm has to be 1 and a factor of $e^{i\varphi}$ makes no difference.

$$a^* \cdot a + b^* \cdot b = 1$$

$$(e^{i\phi})^* \cdot (e^{i\phi}) = 1$$

With two electrons

Two complex numbers are use to describe the spin of one electron so four are used for two.

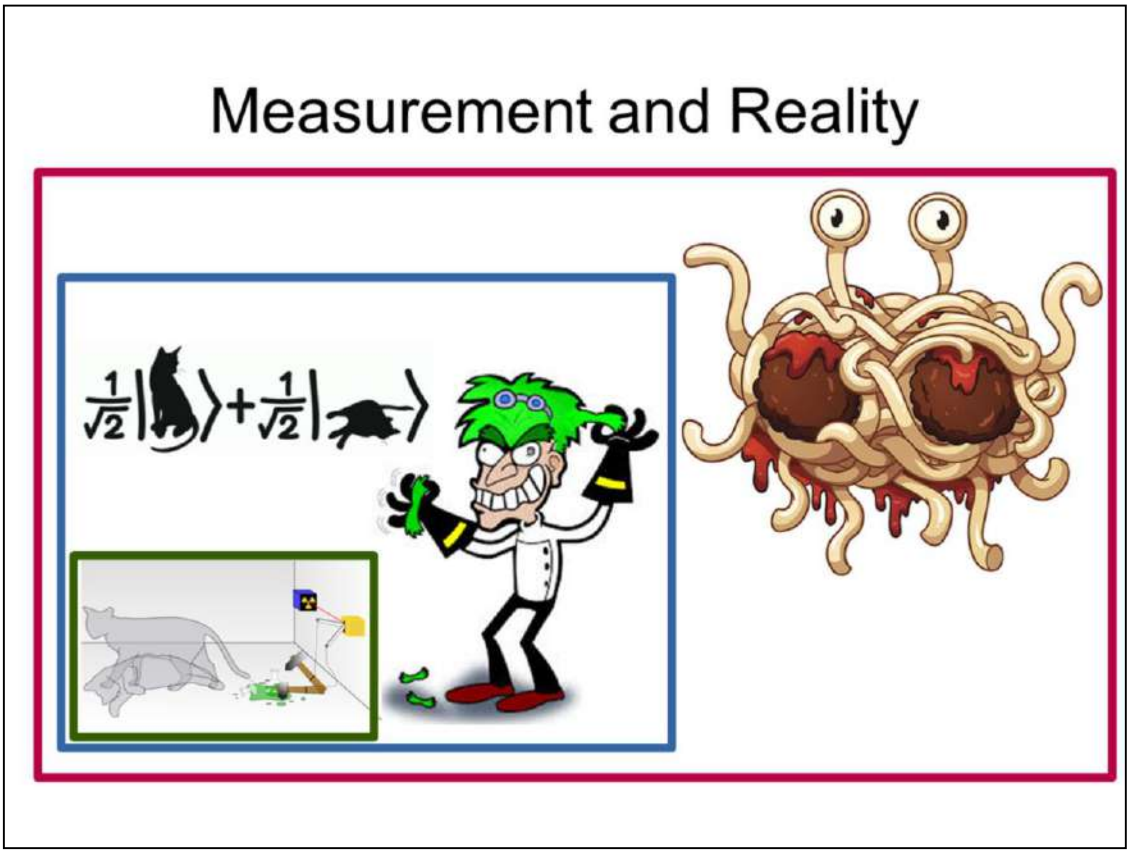
But the normalization and phase only take away two real parameters so there are six left rather than 4.

What are those two extra ones doing? The elctrons may be entangled and can't be fully described separately.

Measurement

- The quantium state should describe everything that will ever happen to something – but measurement gets a particular value.
- Measurement probabilities are given by the square of the norm of the eigenvalues, I.e. $\lambda^* \lambda$ of the matrices applied to the original state.
- Matrices are Unitary $U^\dagger U = 1$
  this preserves the norm 1 and keeps the eigenvectors perpendicular to each other.

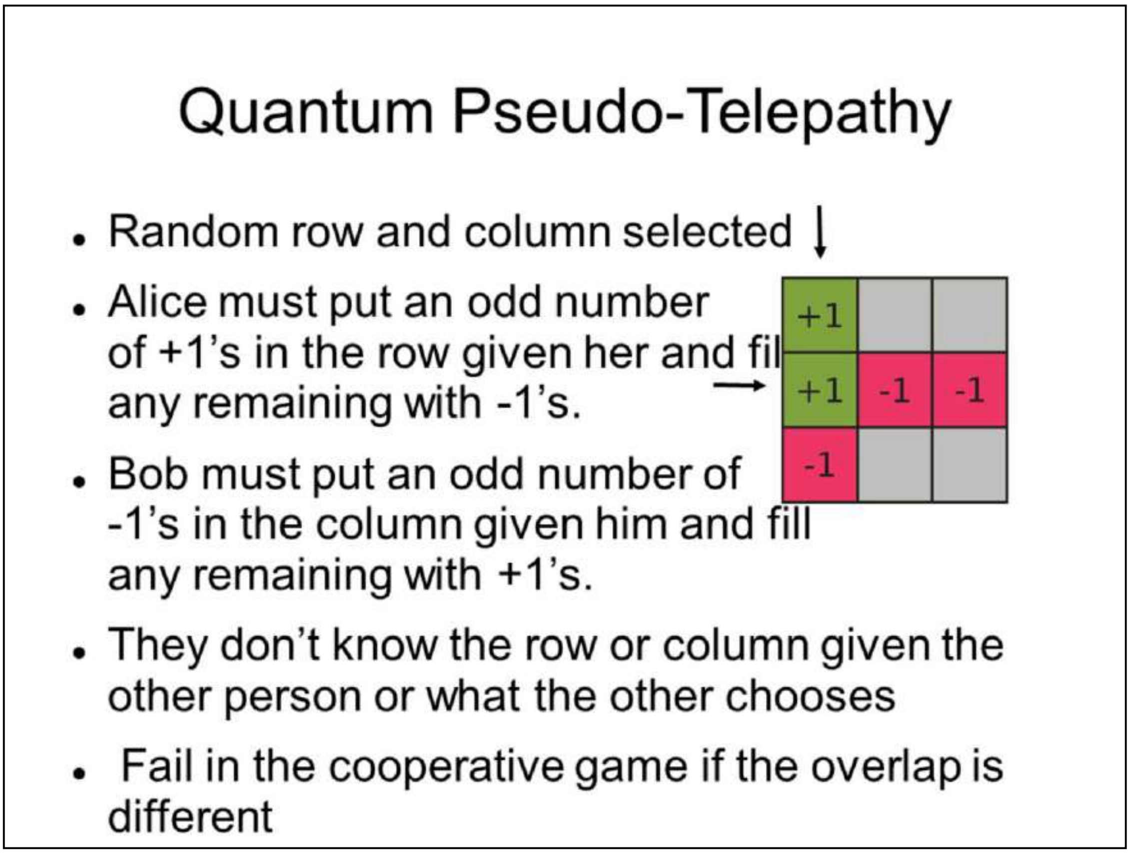
The matrices describe things that can be done without requiring a measurement and ‘collapsing’ the wave function. Like turning the detector for electron spin around 45°.



According to the flying spaghetti monster the evil scientist is in exactly the same sort of superposition as the cat.

The scientist may perform a mesaurment which ‘collapses’ the wave function – but the flying spaghetti monster sees no collapse at that point – the scientist is a superposition of having made the different possible measurements.

There is no such thing as ‘reality’ or a collapse of the wavefunction withing quantum mechanics itself.



An example bringing it all together and introducing Quantum computers

Alice was given row 2 to fill and Bob was given column 1 to fill. They both chose a green for the intersection so they have won the cooperative game.

Classical strategy

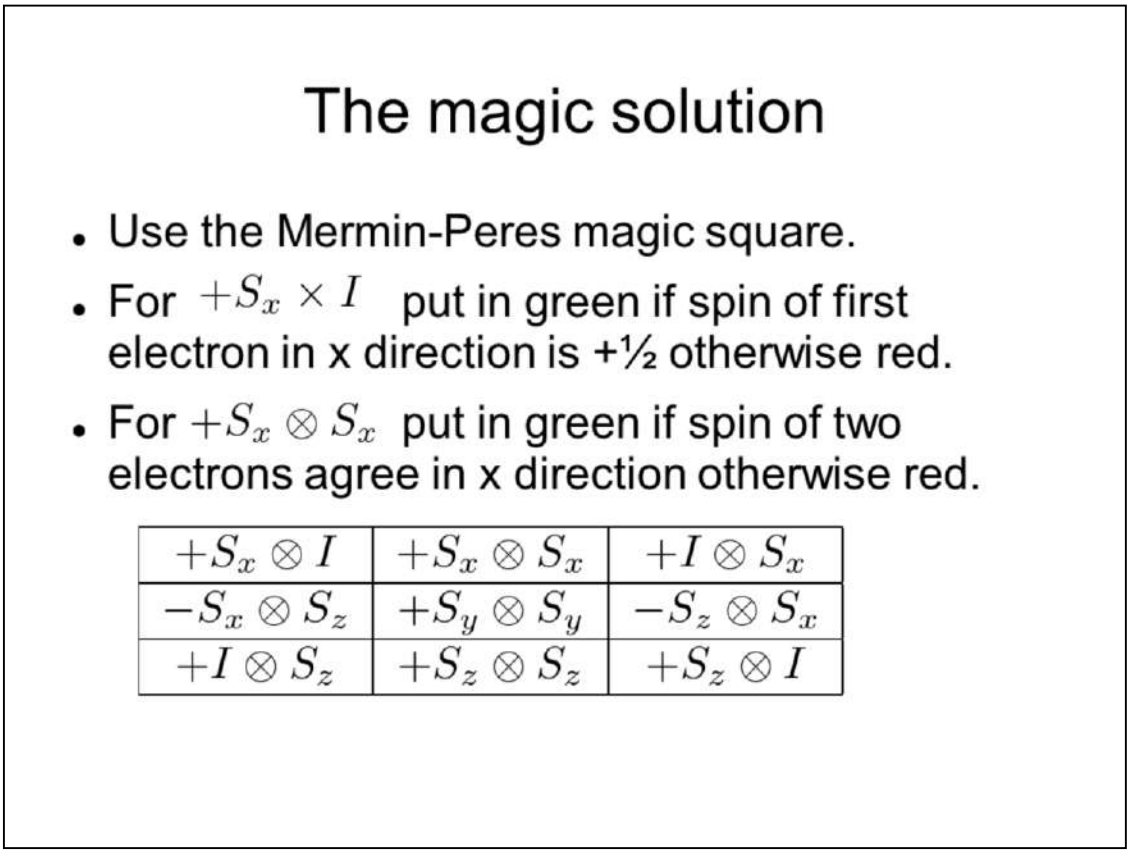
- Alice and Bob can decide on a strategy in advance.
- At least 1/9 chance of failing using any classical strategy
- Alice – an even number of red
  Bob – an even numer of green

But overall an odd number of squares

+1	+1	+1
+1	-1	-1
-1	+1	?

Quantum strategy

- Each go of the game needs a pair of 'Bell pairs' of entangled electrons. Each player takes away two electrons, one from each pair.
- Dependent on the row or column they are given they do the corresponding measurements for that row or column and set green or red correspondingly.
- Not yet done in practice but should be possible eventually.



Can test if both are in the same direction without knowing it for either – like putting two magnets together. Otherwise we would not be able to do this for x y and z as doing dong one for an electron disturbs it for the othes.

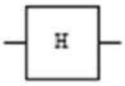
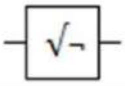
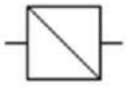
The third measurement for either is redundant – the first two detemine what the value of the third would give.

There is an interaction between measurements in different directios that isn't altogether obvious.

Quantum Computer Gates

Each line represents what happens to a 'Bell pair' with two states like an electron spin.

1-Wire Gates

Name	Hadamard	Square Root of Not	Splitter
Symbol			
Matrix	$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & i\\ i & 1\end{bmatrix}$

From Craig Gidney "Implementing pseudo-telepathy"

Many systems tred in quantum computers, for instance

Ultracold Rydberg atoms – atoms with one or more highly excited electrons and at close to 0 Kelvin.

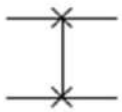
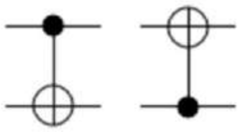
Quantum dots using electron spin

Nuclear spin with nuclear magnetic resonance.

Josephson junctions – superconductors with a thin barrier between them

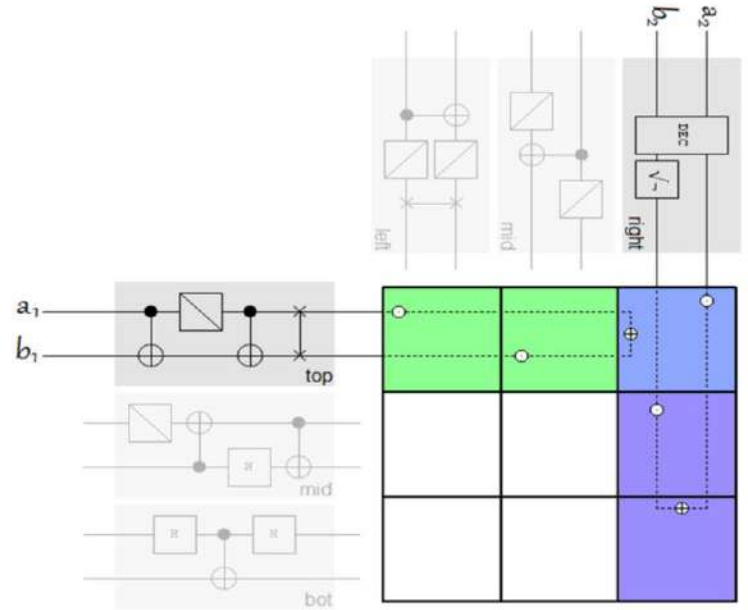
More Gates

2-Wire Gates

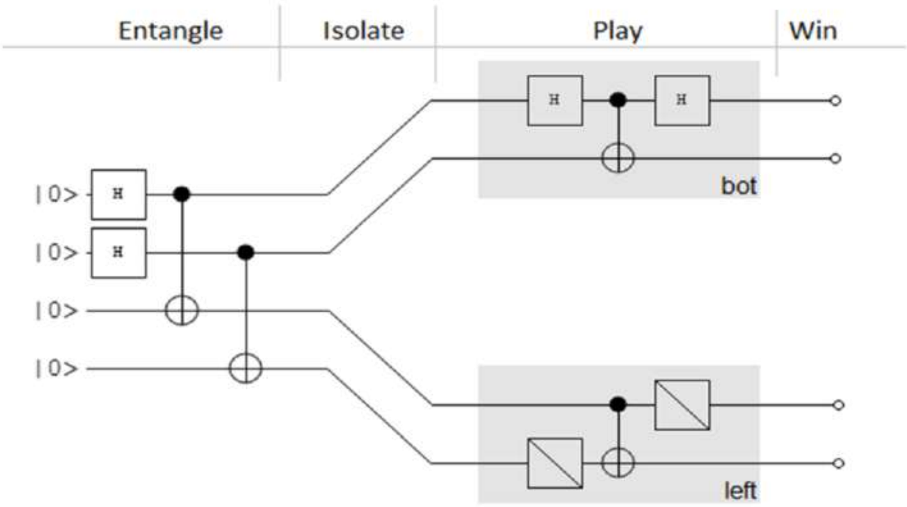
Name	Swap	Controlled Not	Decrement
Symbol			
Matrix	$\begin{bmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{bmatrix}$	$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}$ $\begin{bmatrix}1&0&0&0\\0&0&0&1\\0&0&1&0\\0&1&0&0\end{bmatrix}$	$\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\1&0&0&0\end{bmatrix}$

Board with Row/Col Circuits

Top Row + Right Col Case Highlighted



The whole process



From Craig Gidney "Implementing pseudo-telepathy"

An example

- After entangement have
  $$\frac{1}{2}\left(|0000\rangle + |0101\rangle + |1010\rangle + |1111\rangle\right)$$
- The effect of applying
  $$A_{bottom} \otimes B_{left}$$
- is $\dfrac{1-i}{4}\times$
  $$(i\,|1000\rangle - |0100\rangle - |1010\rangle + i\,|0110\rangle +$$
  $$i\,|0001\rangle - |1101\rangle - |0011\rangle + i\,|1111\rangle$$

Example continued

- Suppose measurement gives the first one

  $|1000\rangle$

- This is 00 for Alice and 10 for Bob
  Alice puts 0, 0, 1-(0 eor 0) in bottom row

Bob puts 0, 1, 0 eor 1 in left column

- Left bottom is 1 in both with an even number of 0's for Alice and even number of 1's for Bob. This happens for all 8 possibilities.