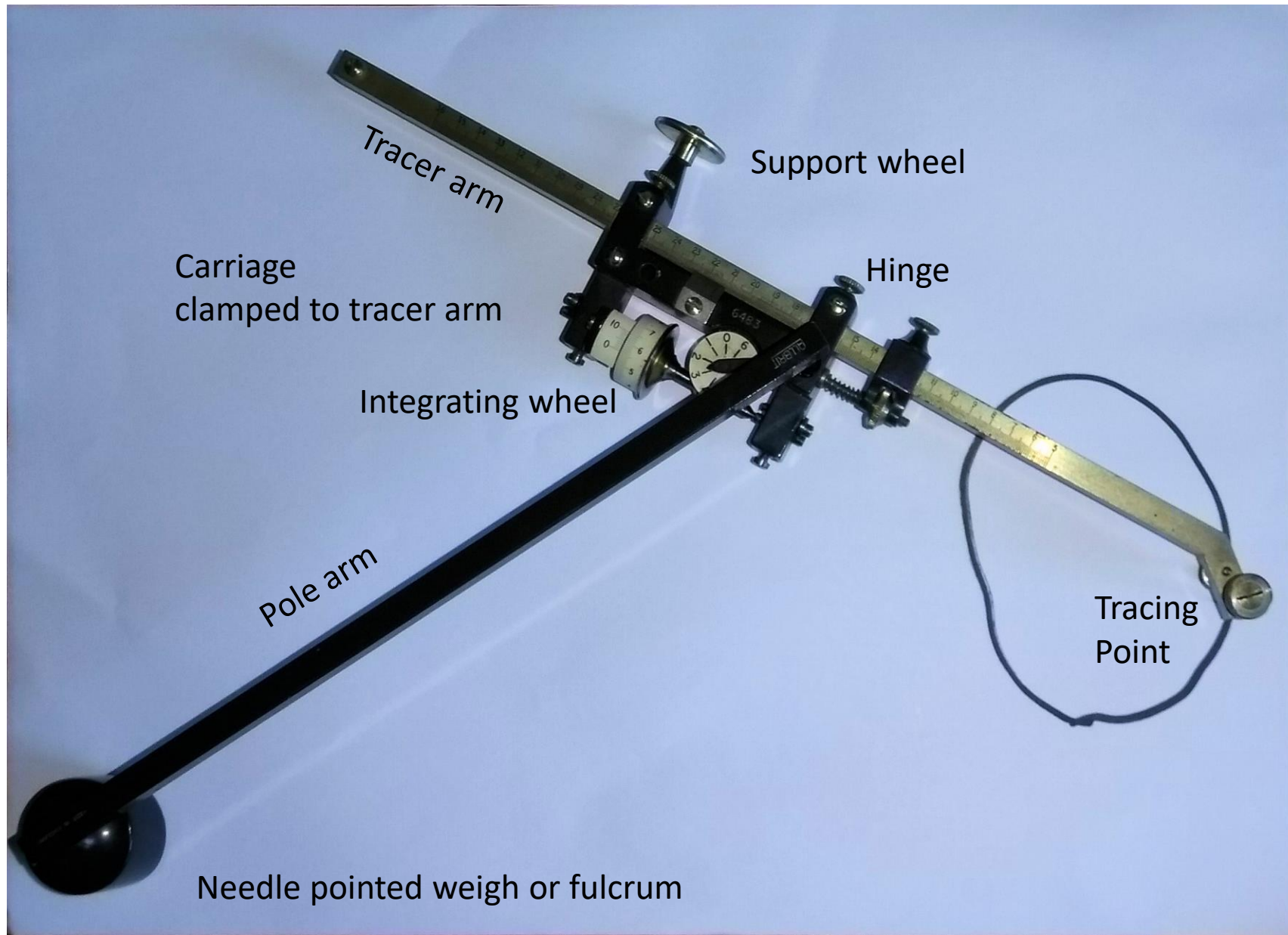# The Planimeter

How complicated a machine is needed to measure an area?

David McQuillan

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Tracer arm

Support wheel

Carriage
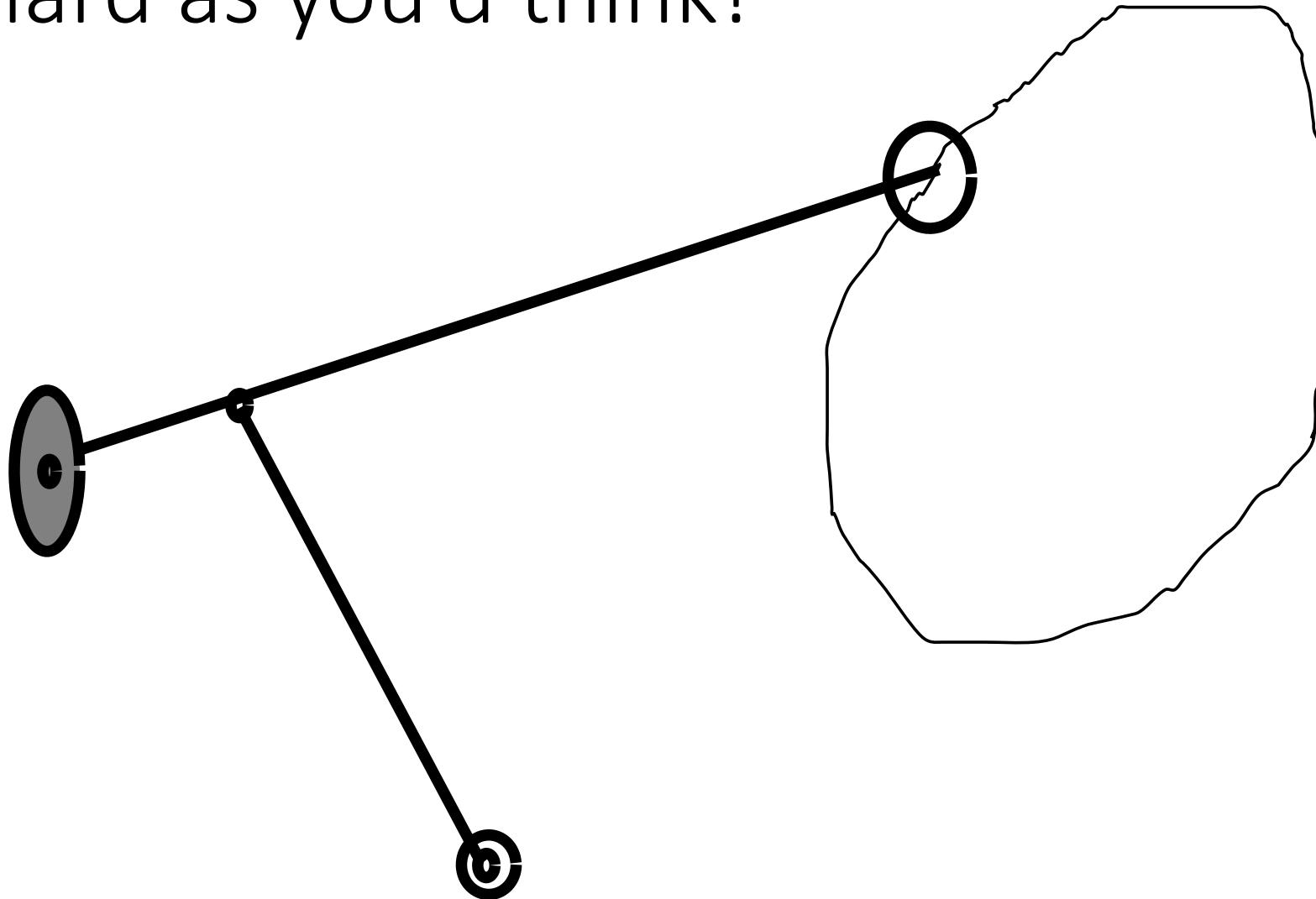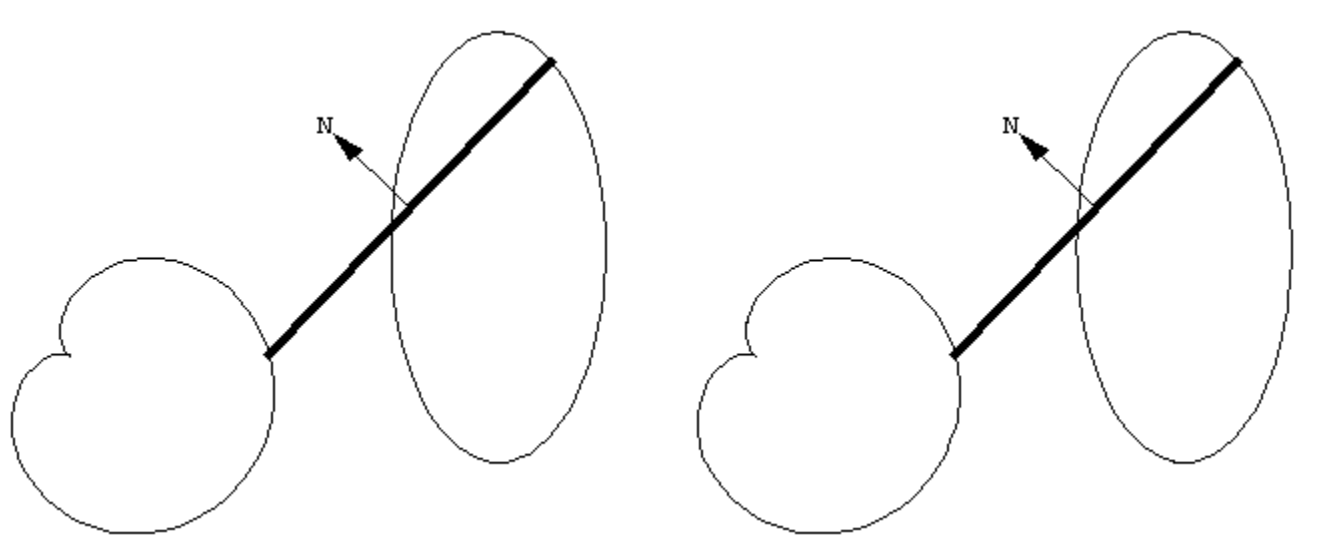clamped to tracer arm

Hinge

Integrating wheel

Pole arm

Tracing
Point

Needle pointed weigh or fulcrum

Not as hard as you'd think!
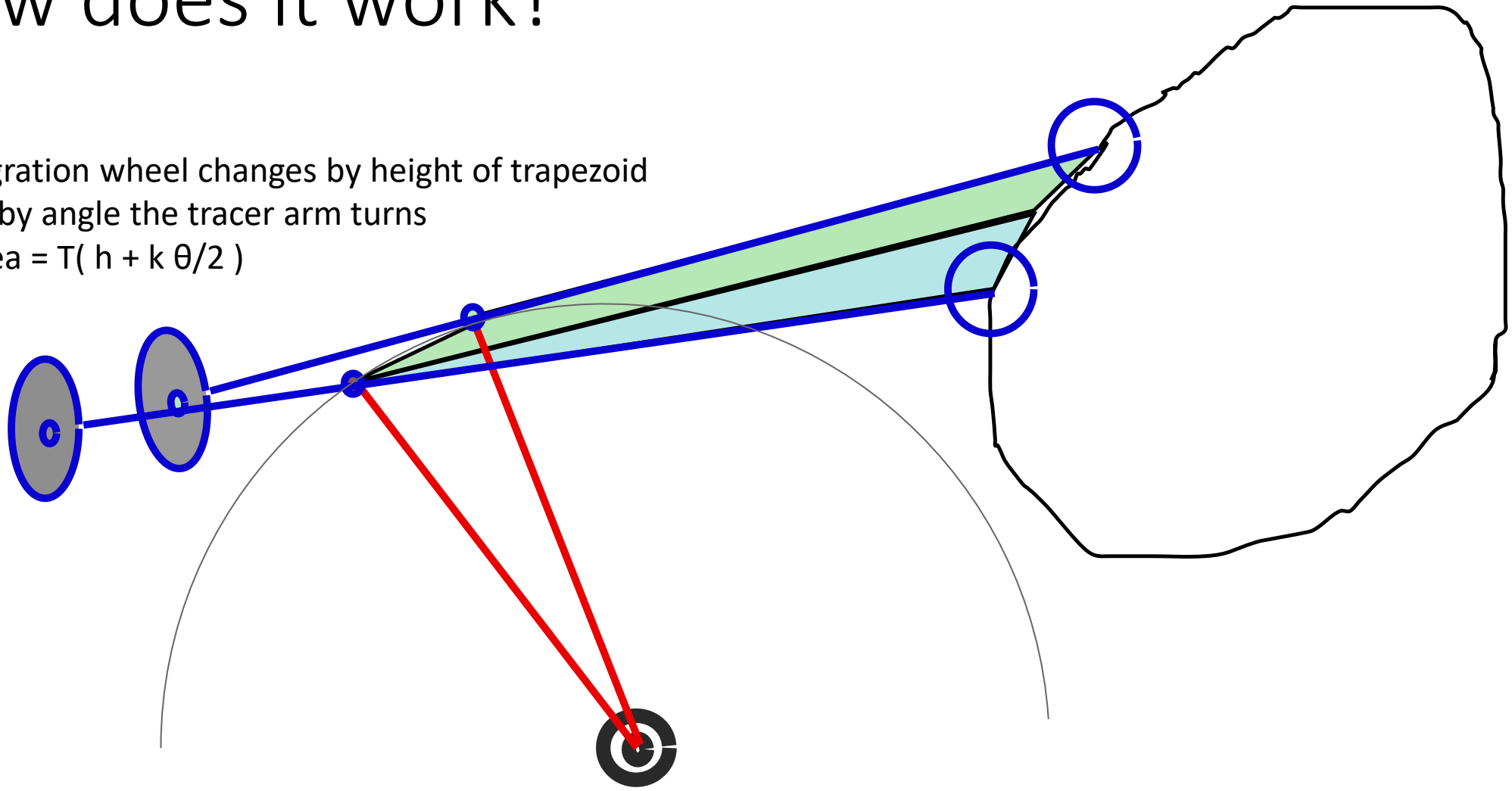
Area difference theorem

The blue area is the positive area and the red the negative area swept out by the segment. The blue area minus the red area gives the area of the right hand side figure minus the left hand side figure.
Can use Green's Theorem but that wasn't discovered till later!

How does it work?

Integration wheel changes by height of trapezoid
plus by angle the tracer arm turns
Δ area = T(h + k θ/2)

How can that work!

- The position of the planimeter is the same after tracing around a shape as when it starts.

- If k=1 the the area is given by a plus area of the far side minus the area on the near side, everything outside the shape disappears.

- Since the angle is the same at the end k doesn't matter!, the measurement on the wheel will cancel out which means the tracer arm length can be changed without worrying about the position of the integration wheel on it

- Any shape could be used to constrain the tracer arm!

History

- Concept was first thought of by Johann Martin Hermann in 1814
- Modern form first built in 1854 by Jakob Amsler-Laffon, a Swiss mathematician who set up a factory producing them.
- Nowadays one can get them with electronics replacing the measuring wheel.

Polar planimeter

Linear planimeter